

$$1. \left. \begin{array}{l} a.x = b.y = c.z = 30 \\ \frac{x+y}{x.y} + \frac{y+z}{y.z} + \frac{x+z}{x.z} = 6 \end{array} \right\} \Rightarrow a + b + c = ?$$

$$ax = by = cz = 30 \Rightarrow x = \frac{30}{a}, y = \frac{30}{b}, z = \frac{30}{c}$$

$$\frac{x+y}{x.y} + \frac{y+z}{y.z} + \frac{x+z}{x.z} = \frac{x}{x.y} + \frac{y}{x.y} + \frac{y}{y.z} + \frac{z}{y.z} + \frac{x}{x.z} + \frac{z}{x.z}$$

$$= \frac{1}{y} + \frac{1}{x} + \frac{1}{z} + \frac{1}{y} + \frac{1}{z} + \frac{1}{x}$$

$$= 2 \cdot \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) = 6$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 3$$

$$\Rightarrow \frac{a}{30} + \frac{b}{30} + \frac{c}{30} = 3$$

$$\Rightarrow \frac{a+b+c}{30} = 3$$

$$\Rightarrow \boxed{a+b+c=90}$$

Cevap: E

$$2. \frac{x-y}{2} = \frac{y+z}{3} = \frac{z}{4} = k \text{ olsun.}$$

$$\Rightarrow \begin{cases} x-y = 2k \\ y+z = 3k \\ \boxed{z = 4k} \end{cases}$$

$$y+z = 3k \rightarrow y+4k = 3k \rightarrow \boxed{y = -k}$$

$$x-y = 2k \rightarrow x+k = 2k \rightarrow \boxed{x = k}$$

$$\Rightarrow 2x - 3y - z = 7$$

$$\Rightarrow 2k + 3k - 4k = 7$$

$$\Rightarrow k = 7$$

$$\Rightarrow \boxed{x = 7}$$

Cevap: D

$$3. \frac{a}{-2} = \frac{b}{-3} = \frac{c}{-4} = k \Rightarrow a = -2k, b = -3k, c = -4k$$

$$a - b + c = 3 \Rightarrow -2k + 3k - 4k = 3 \Rightarrow -3k = 3 \Rightarrow \boxed{k = -1}$$

$$\Rightarrow a = 2, b = 3, c = 4$$

$$\Rightarrow \frac{b.c}{a} = \frac{3.4}{2} = \boxed{6}$$

Cevap: B

$$4. \frac{a}{2} = \frac{b}{3} = \frac{c}{5} = k \Rightarrow a = 2k, b = 3k, c = 5k$$

$$2a + 3b - c = 16 \Rightarrow 2.2k + 3.3k - 5k = 16$$

$$\Rightarrow 4k + 9k - 5k = 16$$

$$\Rightarrow 8k = 16 \Rightarrow \boxed{k = 2}$$

$$\Rightarrow a - 2b + c = 2k - 6k + 5k = k = \boxed{2}$$

Cevap: A

$$5. a, b, c \in \mathbb{Z}^+$$

$$\frac{b}{2} = 3c \Rightarrow b = 6c$$

$$7a + 2b = 69 \Rightarrow 7a + 12c = 69$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ 3 \quad 4 \end{array}$$

$$c = 4, a = 3 \text{ için eşitlik sağlanır.}$$

$$\min(a+c) = 4 + 3 = \boxed{7}$$

Cevap: C

$$6. \left. \begin{array}{l} a + \frac{1}{b} = 2 \Rightarrow a.b + 1 = 2b \Rightarrow a.b = 2b - 1 \\ b + \frac{2}{a} = 3 \Rightarrow a.b + 2 = 3a \Rightarrow a.b = 3a - 2 \end{array} \right\}$$

$$\Rightarrow 2b - 1 = 3a - 2$$

$$\Rightarrow \boxed{3a = 2b + 1}$$

$$\Rightarrow \frac{3a + 2b - 1}{4b} = \frac{2b + \cancel{x} + 2b - \cancel{x}}{4b} = \frac{4b}{4b} = \boxed{1}$$

Cevap: A

7. $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = 2 \Rightarrow x = 2a, y = 2b, z = 2c$

$$\begin{aligned} a.x + b.y + c.z = 5 &\Rightarrow a.2a + b.2b + c.2c = 5 \\ &2a^2 + 2b^2 + 2c^2 = 5 \\ &2.(a^2 + b^2 + c^2) = 5 \\ &\boxed{a^2 + b^2 + c^2 = \frac{5}{2}} \end{aligned}$$

Cevap: C

8. $ax = by = cz = \frac{1}{6} \Rightarrow a = \frac{1}{6x}, b = \frac{1}{6y}, c = \frac{1}{6z}$

$$a + b + c = 30 \Rightarrow \frac{1}{6x} + \frac{1}{6y} + \frac{1}{6z} = 30$$

$$\Rightarrow \frac{y.z + x.z + x.y}{6.x.y.z} = 30$$

$$\Rightarrow \frac{x.y + y.z + x.z}{12} = 30$$

$$\Rightarrow x.y + y.z + x.z = \boxed{360}$$

Cevap: D

9. $a + \frac{3}{b} = 7 \Rightarrow a.b + 3 = 7b \Rightarrow a.b = 7b - 3$

$$2b + \frac{5}{a} = 8 \Rightarrow 2a.b + 5 = 8a \Rightarrow a.b = \frac{8a - 5}{2}$$

$$7b - 3 = \frac{8a - 5}{2}$$

$$14b - 6 = 8a - 5$$

$$14b - 8a = 1$$

$$\boxed{7b - 4a = \frac{1}{2}}$$

$$4a + \frac{1}{2}$$

$$\frac{\textcircled{7b} + 4a - \frac{1}{2}}{2a} = \frac{4a + \frac{1}{2} + 4a - \frac{1}{2}}{2a} = \frac{8a}{2a} = \boxed{4}$$

Cevap: B

10. $\frac{y}{3} = \frac{x}{5} \Rightarrow x = 5k, y = 3k$

$$6x + 7y = 34 \Rightarrow 6.5k + 7.3k = 34$$

$$30k + 21k = 34$$

$$51k = 34$$

$$k = \frac{34}{51} = \frac{2}{3}$$

$$\Rightarrow x - y = 5k - 3k = 2k = 2 \cdot \frac{2}{3} = \boxed{\frac{4}{3}}$$

Cevap: C

11. $\frac{a}{b} = \frac{c}{d} = 12 \Rightarrow a = 12b, c = 12d$

$$\frac{a - b - c + d}{b - d} = \frac{12b - b - 12d + d}{b - d} = \frac{11b - 11d}{b - d}$$

$$= \frac{11.(b - d)}{b - d}$$

$$= \boxed{11}$$

Cevap: B

12. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{1}{5} \Rightarrow b = 5a, d = 5c, f = 5e$

$$\boxed{2a - 3c + 5e = -8}$$

$$2b + 5f = 35 \Rightarrow 2.5a + 5.5e = 35 \Rightarrow \boxed{10a + 25e = 35}$$

$$\begin{array}{r} -5/ \quad 2a - 3c + 5e = -8 \\ \hline 10a + 25e = 35 \end{array}$$

$$\begin{array}{r} -10a + 15c - 25e = 40 \\ + \quad 10a + 25e = 35 \\ \hline 15c = 75 \end{array}$$

$$\boxed{c = 5}$$

$$\Rightarrow d = 5c = 5.5 = \boxed{25}$$

Cevap: E

13. $bc \neq 0$

$$\left. \begin{aligned} \frac{a+3b}{b} = \frac{7}{3} &\Rightarrow 3a+9b=7b \Rightarrow \boxed{3a=-2b} \\ \frac{a+c}{c} = \frac{1}{3} &\Rightarrow 3a+3c=c \Rightarrow \boxed{3a=-2c} \end{aligned} \right\} \Rightarrow b=c$$

$$\frac{a-c}{b-a} = \frac{a-c}{c-a} = \boxed{-1}$$

Cevap: B

14. $\frac{a+2b}{b} = \frac{7}{3} \Rightarrow 3a+6b=7b \Rightarrow 3a=b$

$\frac{a-2c}{c} = \frac{1}{3} \Rightarrow 3a-6c=c \Rightarrow 3a=7c$

$$\frac{a+c}{b-a} = \frac{a+\frac{3a}{7}}{3a-a} = \frac{\frac{10a}{7}}{2a} = \frac{5}{7} \cdot \frac{1}{2} = \boxed{\frac{5}{7}}$$

Cevap: C

15. $a, b, c \in \mathbb{Z}^+$

$2a = b \Rightarrow 5 \cdot \frac{a}{b} = \frac{1}{2} \Rightarrow \frac{a}{b} = \frac{5}{10}$

$5c = 4b \Rightarrow 2 \cdot \frac{c}{b} = \frac{4}{5} \Rightarrow \frac{c}{b} = \frac{8}{10}$

$a = 5k, b = 10k, c = 8k$

$\boxed{a < c < b}$

Cevap: B

16. x ve y ; 5 ve 7 ile ters orantılı ise $x = \frac{k}{5}$, $y = \frac{k}{7}$

$3x + 9y = 99 \Rightarrow 3 \cdot \frac{k}{5} + 9 \cdot \frac{k}{7} = 99 \Rightarrow \frac{2ik+45k}{35} = 99$

$\Rightarrow \frac{66k}{35} = 99 \Rightarrow \boxed{k = \frac{3 \cdot 35}{2}}$

$x - y = \frac{k}{5} - \frac{k}{7} = \frac{2k}{35} = 2 \cdot \frac{3 \cdot 35}{2} \cdot \frac{1}{35} = \boxed{3}$

Cevap: E

17. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{1}{4} \Rightarrow b = 4a, d = 4c, f = 4e$

$\boxed{2a - 3c + e = 5}$

$3d - f = 16 \Rightarrow 3 \cdot 4c - 4e = 16 \Rightarrow \boxed{12c - 4e = 16}$

$$\begin{array}{r} 4/ \quad 2a - 3c + e = 5 \\ \quad \quad 12c - 4e = 16 \\ \hline \end{array} \quad \begin{array}{r} 8a - 12c + 4e = 20 \\ + \quad 12c - 4e = 16 \\ \hline 8a = 36 \\ a = \frac{36}{8} \\ = \boxed{\frac{9}{2}} \end{array}$$

Cevap: A

18. $\frac{1}{a} + \frac{1}{b} = 15$

$\frac{1}{b} + \frac{1}{c} = 17$

$+ \frac{1}{a} + \frac{1}{c} = 22$

$$2 \cdot \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = 44 \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 22$$

$$\underbrace{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}_{15} = 22$$

$$\Rightarrow \frac{1}{c} = 7$$

$$\Rightarrow \boxed{c = \frac{1}{7}}$$

Cevap: C

19. $n \in \mathbb{N}$

$$\left. \begin{array}{l} a = 2^n + 6^n \\ b = 6^n - 2^n \end{array} \right\} \Rightarrow \frac{a}{b} = \frac{2^n + 6^n}{6^n - 2^n} = \frac{14}{13}$$

$$13 \cdot 2^n + 13 \cdot 6^n = 14 \cdot 6^n - 14 \cdot 2^n$$

$$27 \cdot 2^n = 6^n$$

$$3^3 \cdot 2^{3n} = 2^n \cdot 3^n$$

$$3^3 = 3^n \Rightarrow \boxed{n=3}$$

$$\begin{aligned} a - b &= (2^n + 6^n) - (6^n - 2^n) = 2^n + 6^n - 6^n + 2^n = 2 \cdot 2^n \\ &= 2 \cdot 2^3 \\ &= 2^4 \\ &= \boxed{16} \end{aligned}$$

Cevap: B

20.

$$\frac{K}{L} = \frac{1}{7}$$

$$7/ \left. \begin{array}{l} \frac{L}{M} = \frac{1}{4} \Rightarrow \frac{L}{M} = \frac{7}{28} \end{array} \right\} \Rightarrow \begin{array}{l} K = k \\ L = 7k \\ M = 28k \end{array}$$

$$K + L + M = 72 \Rightarrow k + 7k + 28k = 72$$

$$\Rightarrow 36k = 72$$

$$\Rightarrow k = 2$$

$$\Rightarrow L = 7k = 7 \cdot 2 = \boxed{14}$$

Cevap: C